ANTIMAXIMUM PRINCIPLE AND FUČIK SPECTRUM

Jean-Pierre GOSSEZ
Universidad Libre de Bruselas

Let us consider the linear problem

$$Lu = \lambda u + h(x) \text{ in } \Omega,\ B(u) = 0 \text{ on } \partial\Omega, \tag{1}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, L is a second order linear symmetric elliptic operator in divergence form on Ω and $B(u)$ denotes the boundary conditions. Let λ_1 be the first eigenvalue of L under the above boundary conditions.

In this context the antimaximum principle asserts that given $h \geq 0$, $h \not\equiv 0$, there exists $\delta = \delta(h) > 0$ such that if $\lambda \in]\lambda_1, \lambda_1 + \delta[$, then any solution u of (1) satisfies $u < 0$ on $\bar{\Omega}$. Moreover in some cases, δ can be taken independant of h. We say then that the antimaximum principle holds *uniformly* and we denote by δ_1 the largest δ admissible.

It is known there exists a connexion between the antimaximum principle and the behaviour at infinity of the corresponding Fučik spectrum. We recall that this spectrum is defined as the set Σ of those $(\alpha, \beta) \in \mathbb{R}^2$ such that

$$Lu = \alpha u^+ - \beta u^- \text{ in } \Omega, \qquad B(u) = 0 \text{ on } \partial\Omega \tag{2}$$

has a nontrivial solution u. It was shown that Σ is contained in the smaller set $\{(\alpha, \beta) \in \mathbb{R}^2;\ \alpha \text{ and } \beta \geq \lambda_1 + \epsilon\} \cup (\lambda_1 \times \mathbb{R}) \cup (\mathbb{R} \times \lambda_1)$ for some $\epsilon > 0$ if and only if the antimaximum principle holds uniformly. Moreover the largest ϵ admissible here is equal to the number δ_1 introduced before.

The main aim of this course is to present some recent results concerning with this topic and its relation with the set Σ, both in this simple case as in more complicated problem like, for instance,

$$-\Delta_p u = \lambda m(x)|u|^{p-2}u + h(x) \text{ in } \Omega,\ \partial u/\partial\nu = 0 \text{ on } \partial\Omega, \tag{3}$$

where $\Delta_p u = \text{div}(|\nabla u|^{p-2}\nabla u)$, $1 < p < \infty$, is the p-laplacian and $\partial/\partial\nu$ denotes the normal derivation on $\partial\Omega$ and m is a weight function that can change sign. The appearance of indefinite weights cause surprising results.